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APPENDIX: "DYNAMIC EQUILIBRIUM IN A COMPETITIVE CREDIT MARKET: INTER-TEMPORAL CONTRACTING AS INSURANCE AGAINST RATIONING"

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PROOFS FOR

DYNAMIC EQUILIBRIUM IN A COMPETITIVE CREDIT MARKET: INTERTEMPORAL CONTRACTING AS INSURANCE AGAINST RATIONING

by

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APPENDIX

PROOF OF THEOREM 1: Note that each value of z can be viewed as a distinct contract in the Riley (1979) framework. Given this, our definition of a DSINC policy corresponds precisely with the definition of strongly informationally consistent (SINC) policies in the (static) framework of Riley. Now, the only essential difference between the DRE and Riley's reactive equilibrium is the additional subgame perfection requirement in the DRE. However, this requirement appears both in the definition of the DRE and the definition of DSINC policies. Thus, applying Riley's logic, it must be true that the Pareto dominant DSINC policy is the DRE. Q.E.D.

PROOF OF LEMMA 1: The proof involves a comparison of the collateral allocations $\{C_I^H = \hat{C}_I^H;\ C_{III}^H = W\}$ and $\{C_I^H = \hat{C}_I^H - \Phi;\ C_{III}^H = W - \Phi\}$ $\forall\ \Phi \in (0,\ C_{III}^H(\text{opt}) - W]$, where $C_{III}^H(\text{opt})$ is the (optimal) level of collateral needed to avoid rationing and Φ is a real valued (scalar) perturbation. Note that

$C_{III}^H(\text{opt}) = \delta r[\Phi\delta^H]^{-1}$.

We will prove the lemma for the more restrictive case in which borrowers rationed at t=0 get the same second period contracts as the (lucky) good borrowers that obtained credit at t=0. As we know, such an assumption improves the attractiveness of rationing at t=0. Thus, if the lemma holds for this (overly) restrictive case, it must hold in general. Let single hats on variables denote the original solution with no rationing at t=0 and let double hats on variables denote the alternative solution with rationing at t=0. Note that the contracts for the borrowers which are bad at t=0 are the same in both alternatives. This means

$$\hat{\hat{U}}_I(\delta^L|\delta^L) = \hat{U}_I(\delta^L|\delta^L). \tag{A-1}$$

Further, in equilibrium (see (13))

$$U_I(\delta^L|\delta^L) = U_I(\delta^H|\delta^L). \tag{A-2}$$

Combining (A-1) and (A-2) yields

$$\hat{\hat{U}}_I(\delta^H|\delta^L) = \hat{U}_I(\delta^H|\delta^L). \tag{A-3}$$

From the definitions under (2)' and the condition (4)' we got

$$\hat{U}_I(\delta^H|\delta^L) = \delta^L R_N^H - \Phi\hat{C}_I^H - \delta^L[\{1-\mu\}\delta^H + \mu\delta^L]R_N^H - \bar{\delta}^L\delta^L R_N^L. \tag{A-4}$$

Further,

$$\hat{\hat{U}}_I(\delta^H|\delta^L) = \hat{\hat{\pi}}_I^H[\delta^L R_N^H - \Phi\{\hat{C}_I^H - \Phi\}] - \delta^L[\{1-\mu\}\delta^H + \mu\delta^L]R_N^H - \bar{\delta}^L\delta^L R_N^L. \tag{A-5}$$

Substituting (A-4) and (A-5) in (A-1) gives

$$\hat{\hat{\pi}}_I^H = \{\delta^L R_N^H - \Phi\hat{C}_I^H\}\{\delta^L R_N^H - \Phi[\hat{C}_I^H - \Phi]\}^{-1}. \tag{A-6}$$

From (11), (12), (4)' and the definition under (3)', we have

$$\begin{aligned}\hat{\hat{U}}_I(\delta^H|\delta^H) &= \hat{\hat{\pi}}_I^H\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H[C_I^H - \Phi]\} + [\delta^H]^2 R_N^H \\ &\quad + \bar{\delta}^H\nu\delta^L R_N^L - \bar{\delta}^H[1-\nu]\hat{\hat{\pi}}_{III}^H\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H[W-\Phi]\}\end{aligned} \tag{A-7}$$

where $\hat{\hat{\pi}}_{III}^H = \{\delta^L R_N^L\}\{\delta^L R_N^H - \Phi[W+\Phi]\}^{-1}$.

From (A-7), it follows that

$$\begin{aligned}&\partial\hat{\hat{U}}_I(\delta^H|\delta^H)/\partial\Phi \\ &= -\{\Phi[\delta^L R_N^H - \Phi\hat{C}_I^H]\}\{\delta^L R_N^H - \Phi[\hat{C}_I^H - \Phi]\}^{-2}\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H[\hat{C}_I^H - \Phi]\} \\ &\quad + \{\delta^L R_N^H - \Phi\hat{C}_I^H\}\{[1-\beta]\bar{\delta}^H\}\{\delta^L R_N^H - \Phi[\hat{C}_I^H - \Phi]\}^{-1} \\ &\quad - \bar{\delta}^H[1-\nu]\delta^L R_N^L[1-\beta]\bar{\delta}^H\{\delta^L R_N^H - \Phi[W + \Phi]\}^{-1} \\ &\quad + \bar{\delta}^H[1-\nu]\Phi\delta^L R_N^L\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H[W + \Phi]\}\{\delta^L R_N^H - \Phi[W + \Phi]\}^{-1}\end{aligned} \tag{A-8}$$

and $\partial^2\hat{\hat{U}}_I(\delta^H|\delta^H)/\partial\Phi^2$

$$\begin{aligned}&= 2\Phi G_1\{\delta R_N^H - \delta^L\bar{\delta}^H[1-\beta][1-\beta\bar{\delta}^H\{\delta^H\}^{-1}][\hat{C}_I^H - \Phi]\}\{G_1 + \Phi\Phi\}^{-3} \\ &\quad - 2\Phi\delta^L\delta R_N^L R_N^H\bar{\delta}^H[1-\nu]\{\delta^L R_N^H - \Phi[W - \Phi]\}^{-3},\end{aligned} \tag{A-9}$$

where $G_1 \equiv \delta^L R_N^H - \Phi\hat{C}_I^H$.

Note that both terms in (A-9) are strictly positive. Thus, if it is optimal to accept some rationing at t=0 (i.e., the transfer of a small level, Φ, of collateral is positive), then it is always optimal to maximize Φ (i.e., maximize rationing at t=0) such that rationing of good borrowers in the contracts node III is completely eliminated. This tells us that rationing will be restricted to <u>one</u> of the contracts -- either contract I or contract III -- and will <u>not</u> occur in both. Thus, it is left to compare the allocation $\{\hat{C}_I^H = C_I^H;\ C_{III}^H = W\}$ with the allocation $\{C_I^H = \hat{C}_I^H - \Phi_{max};\ C_{III}^H = W - \Phi_{max}\}$, where $\Phi_{max} = C_{III}^H(opt) - W$. The original allocation implies $\hat{\pi}_1^H = 1$, $\hat{\pi}_{III}^H = \delta^L R_N^L\{\delta^L R_N^H - \Phi W\}^{-1}$. (This follows from (A-6) and (A-7) for $\Phi = 0$.) Note that π_{III}^H equals one for an optimally collaterized contract. Thus, we have

$$\pi_{III}^H\Big|_{C_{III}^H = C_{III}^H(opt)} = \delta^L R_N^L\{\delta^L R_N^H - \Phi C_{III}^H(opt)\}^{-1} = 1,$$

which implies

$$\delta^L R_N^L = \delta^L R_N^H - \Phi C_{III}^H(opt).$$

This allows us to rewrite the original solution as

$$\hat{\pi}_I^H = 1,\quad \hat{\pi}_{III}^H = \{\delta^L R_N^H - \Phi C_{III}^H(opt)\}\{\delta^L R_N^H - \Phi W\}^{-1}.$$

The alternative solution follows directly from (A-6) and (A-7) for

$$\Phi = \Phi_{max} \equiv C_{III}^H(opt) - W.$$ It follows that

$$\hat{\hat{\pi}}_1^H = \{\delta^L R_N^H - \Phi\hat{C}_I^H\}\{\delta^L R_N^H - \Phi\hat{C}_I^H + \Phi[C_{III}^H(opt) - W]\}^{-1},$$

$$\hat{\pi}_{III}^H = 1.$$

Now, we want to show that

$$\hat{U}_I(\delta^H|\delta^H) - \hat{\hat{U}}_I(\delta^H|\delta^H)\Big|_{\Phi = C_{III}^H(opt) - W} \geq 0.$$

Note that

$$\hat{U}_I(\delta^H|\delta^H) = \hat{\hat{U}}_I(\delta^H|\delta^H)\Big|_{\Phi = 0}$$

Using (A-8) gives us

$$\hat{U}_I(\delta^H|\delta^H) - \hat{\hat{U}}_I(\delta^H|\delta^H)\Big|_{\Phi = C^H_{III}(opt) - W}$$

$$=[\hat{\pi}^H_I - \hat{\hat{\pi}}^H_I][\delta^H R^H_N - \bar{\delta}^H\{1-\beta\}\hat{C}^H_I] - \hat{\hat{\pi}}^H_I\bar{\delta}^H[1-\beta][C^H_{III}(opt) - W]$$
$$- \bar{\delta}^H[1-\nu][\hat{\pi}^H_{III} - \hat{\hat{\pi}}^H_{III}][\delta^H R^H_N - \{1-\beta\}\bar{\delta}^H W]$$
$$+ \bar{\delta}^H[1-\nu]\hat{\hat{\pi}}^H_{III}[1-\beta]\bar{\delta}^H[C^H_{III}(opt) - W].$$

Making the appropriate substitutions for all of the credit granting probabilities yields

$$\hat{U}_I(\delta^H|\delta^H) - \hat{\hat{U}}_I(\delta^H|\delta^H)\Big|_{\Phi = C^H_{III}(opt) - W}$$

$$= [\hat{C}^H_{III} - W]R^H_N[\Phi\delta^H - \bar{\delta}^H\{1-\beta\}\delta^L][G_2^{-1} - G_3] \tag{A-10}$$

where $G_2 \equiv \delta^L R^H_N - \Phi\hat{C}^H_I - \Phi[C^H_{III}(opt) - W]$

$G_3 \equiv \bar{\delta}^H[1-\nu][\delta^L R^H_N - \Phi W]^{-1}$.

The expression in (A-10) is positive if

$$G_2^{-1} > G_3.$$

which means we need

$$\delta^L R^H_N - \Phi W > \bar{\delta}^H[1-\nu][\delta^L R^H_N - \Phi\hat{C}^H_I - \Phi\{C^H_{III}(opt) - W\}],$$

or equivalently

$$[1-\bar{\delta}^H\{1-\nu\}]\delta^L R^H_N > \Phi[1-\bar{\delta}^H\{1-\nu\}]W + \bar{\delta}^H[1-\nu]\Phi C^H_{III}(opt) - \bar{\delta}^H[1-\nu]\Phi. \tag{A-11}$$

The above inequality can be made more restrictive by substituting more restrictive values for R, W and $\hat{C}^H_I$.

Take

$R \geq r[\delta^L]^{-1}$, $W \leq C^H_{III}(\text{opt})$ and $\hat{C}^H_I \geq \delta r[\Phi\delta^H]^{-1}[1+\delta^L\{1-\beta\}\bar{\delta}^H\Phi^{-1}]$,

where the last inequality follows from (15). Note that if (A-11) holds with the above inequalities substituted in, then it must hold in general (when the above inequalities do not hold). This is because the above inequalities make it more difficult to satisfy (A-11). Also substituting $C^H_{III}(\text{opt}) = \delta r[\Phi\delta^H]^{-1}$ gives us the following version of (A-11)

$$[1-\bar{\delta}^H\{1-\nu\}]\delta^L[r\{\delta^L\}^{-1} - r\{\delta^H\}^{-1}] > G_4 + G_5,$$

where $G_4 \equiv [1-\bar{\delta}^H\{1-\nu\}]\delta r[\delta^H]^{-1}$

$$G_5 \equiv \bar{\delta}^H[1-\nu]\Phi\delta r[\Phi\delta^H]^{-1} - \bar{\delta}^H[1-\nu]\Phi\delta r[1+\delta^L\{1-\beta\}\bar{\delta}^H\Phi^{-1}][\Phi\delta^H]^{-1}$$

Thus, we need

$$G_4 > G_4 - \bar{\delta}^H[1-\nu]\delta^L[1-\beta]\bar{\delta}^H,$$

or

$$-[\bar{\delta}^H]^2[1-\nu]\delta^L[1-\beta] < 0,$$

which is certainly true. Thus, we have shown that

$$\hat{U}_I(\delta^H|\delta^H) - \hat{U}_I(\delta^H|\delta^H)\Big|_{\Phi = C^H_{III}(\text{opt}) - W} > 0,$$

implying that rationing at t=0 is not optimal. Q.E.D.

PROOF OF THEOREM 2: Obvious, since the theorem is only a collection of results established in the proof of Lemma 1 and elsewhere. Q.E.D.

PROOF OF LEMMA 2: It is sufficient to show that (16) holds for $C^H_{III} = 0$ and $C^H_{III} = C^H_{III}(\text{opt})$. For $C^H_{III} = 0$, (16) becomes

$$\delta^H[\delta^L]^{-1} \geq \delta^H[\delta^L]^{-1},$$

and for $C^H_{III} = \delta r[\Phi\delta^H]^{-1}$, (16) becomes

$$\{\delta^H R - r - [1-\beta]\bar{\delta}^H \delta r[\phi\delta^H]^{-1}\}\{\delta^L R - \delta^L r[\delta^H]^{-1} - \delta r[\delta^H]^{-1}\}^{-1} \geq \delta^H[\delta^L]^{-1},$$

which holds IFF

$$\{1+[1-\beta]\bar{\delta}^H \delta[\phi\delta^H]^{-1}\}r\{\delta^H[\delta^L]^{-1}\}^{-1} \leq 1.$$

The above inequality can be written as

$$\delta^L\delta^H\phi + [1-\beta]\bar{\delta}^H\delta\delta^L \leq [\delta^H]^2\phi,$$

or equivalently

$$\delta^L\delta^H\bar{\delta}^L - \beta[\delta^L]^2\bar{\delta}^H + \bar{\delta}^H\delta^L\delta - \beta\bar{\delta}^H\delta^L\delta \leq [\delta^H]^2\bar{\delta}^L - \beta\delta^L\delta^H\bar{\delta}^H,$$

which holds if

$$\beta\delta^L\bar{\delta}^H\delta - \beta\bar{\delta}^H\delta^L\delta < \delta^H\bar{\delta}^L\delta - \delta^L\bar{\delta}^H\delta,$$

or

$$0 < \delta^2,$$

which is certainly true. This establishes (16). Q.E.D.

PROOF OF THEOREM 3: Note that rationing only occurs in the Contracts III nodes. We know from our previous analysis that in those nodes the following equality holds

$$U^L_{III}(\delta^H|\delta^L) = U^L_{III}(\delta^L|\delta^L).$$

In general this implies,

$$\pi^H_{III}\{\delta^L[R-\alpha^H_{III}] - \bar{\delta}^L C^H_{III}\} = \pi^L_{III}\{\delta^L R^L_N - \bar{\delta}^L[1-\beta]C^L_{III}\}. \quad \text{(A-12)}$$

Our equilibrium concept (more specifically, the sub-game perfection requirement) implies that only one of the contracts III may involve collateral and rationing. This is because rationing and collateral involve deadweight costs, and it is only in the interest of bank and borrower to bear those costs if they are necessary for separation. For example, if both contracts involve rationing, the bank and the borrower can increase their payoffs by removing rationing in one of these contracts and reducing it in the other contract.

without losing separation. Note also it is the contract for good borrowers that involves collateral and, if necessary, rationing. Therefore, we may rewrite (A-12) as,

$$\pi^H_{III}\{\delta^L[R-\alpha^H_{III}] - \bar{\delta}^L C^H_{III}\} = \delta^L[R-\alpha^L_{III}] \qquad \text{(A-13)}$$

Now, we establish that the contract $\{\alpha^H_{III}, C^H_{III}, \pi^H_{III}\}$ should always be a zero-profit contract for the bank. That is, $\delta^H\alpha^H_{III} + \bar{\delta}^H\beta C^H_{III} = r$. Note that if the contract were profitable, an entering (second period) spot market bank would be able to break the "profitable" original contract. Likewise, a more complicated argument asserts that the contract can not impose losses on the bank. We shall present this argument verbally. From Figure 1 we see that the first period contracts are also separating. Notice that these contracts take into account the entire two period time horizon, and the second period contracts have an impact on the incentive compatibility of these first period contracts. Incentive compatibility of the first period contract for good borrowers is easier to establish if the contracts III are as unattractive as possible. The reason is that mimicking bad borrowers at t=0 are more likely than the good borrowers at t=0 to end up in the contracts III nodes. Hence, making the contracts III as unattractive as possible "costlessly" resolves part of the incentive compatibility problem at t=0. Thus, we have

$$\tilde{\alpha}^H_{III} = r[\delta^H]^{-1} - \bar{\delta}^H\beta\tilde{C}^H_{III}[\delta^H]^{-1}. \qquad \text{(A-14)}$$

The intertemporal contract solution to be presented later will indicate that good borrowers in the contracts node III will always use all the collateral they have available. (Note, it is assumed that $W_0 - \tilde{C}^H_I < C^H_{III}(\text{opt})$). That is,

$$\tilde{C}^H_{III} = W_0 - \tilde{C}^H_I. \qquad \text{(A-15)}$$

The arguments used above to determine $\tilde{\alpha}^H_{III}$ in principle also hold for the determination of $\tilde{\alpha}^L_{III}$. That is, incentive compatibility of t=0 contracts ask for $\tilde{\alpha}^L_{III}$ to be as high as possible while entering spot market banks at t=1 force an upper bound on this interest factor of r/δ^L. But look at (A-13). If the equality in (A-13) currently holds for some $\pi^H_{III} < 1$ (that is, rationing occurs), then in order to increase π^H_{III} without disturbing the equality, one should reduce α^L_{III} and/or increase α^H_{III} and/or increase C^H_{III}. Note that an increase in C^H_{III} is ruled out by (A-15) and an increase in α^H_{III} invites competition from entering spot market banks. Hence, its value is fixed by (A-14). Reducing α^L_{III} is a possibility but it will make the incentive compatibility of the contracts at t=0 more difficult. In other words, once we reduce α_{III}, we have to adjust α^H_I and C^H_I in order to preserve incentive compatibility of the first period contracts. By (A-15) this will also affect C^H_{III}. Thus, we see that the following variables must be adjusted in order to remove rationing from contracts node III

$$\{\alpha^H_I,\ C^H_I,\ \pi^H_{III},\ C^H_{III},\ \alpha^L_{III}\}. \tag{A-16}$$

The variables related to the borrower's contract in node I do not enter this set because the intertemporal contract solution will indicate that these are always first best. The contracts node II variables do not enter the set either, because in the intertemporal contract solution a binding constraint will be in effect on those variables.

As argued earlier, rationing might disappear from the model by reducing α^L_{III} in (A-13). This makes the contracts node III more attractive, which causes incentive compatibility problems because mimicking bad borrowers at t=0 are more likely to end up in these contracts. These incentive compatibility problems require an adjustment in $(\alpha^H_I,\ C^H_I)$, which in itself affects the contract for good borrowers in node III since C^H_{III} depends on C^H_I through (A-15). Furthermore, a reduction in α^L_{III} has also a primary effect in that it causes losses to the bank which must be recouped through an offsetting adjustment in (α_I^H, C_I^H). We can summarize this schematically as follows

$$
\begin{tikzcd}
\alpha_{III}^L \downarrow \arrow[rrr, Rightarrow, "\text{(A) by (A-13)}"] \arrow[dr, Rightarrow] & & & \begin{bmatrix} \overset{\approx}{\pi}{}_{III}^H & \uparrow \\ \overset{\approx}{\alpha}{}_I^H & \uparrow \end{bmatrix} \\
\text{(B)} & \begin{bmatrix} C_I^H & \uparrow \\ \alpha_I^H & \downarrow \end{bmatrix} \arrow[r, "\text{by (A-15)}"] & C_{III}^H \uparrow \arrow[r, "\text{by (A-13)}"] & \pi_{III}^H \downarrow
\end{tikzcd}
$$

where "≈" distinguishes the effects (A) and (B):

(A) <u>direct effect</u>: $\overset{\approx}{\pi}{}_{III}^H$ increases by (A-13) and $\overset{\approx}{\alpha}{}_I^H$ increases by the intertemporal zero expected profit condition for the bank;

(B) <u>indirect effect</u>: C_I^H increases and α_I^H decreases, with both adjustments chosen such that it has no effect on the expected profit of the bank, and it makes first period contracts more incentive compatible. The effect of C_I^H on C_{III}^H and π_{III}^H follows directly from (A-15) and (A-13).

Now, we will derive the sufficiency condition for no rationing in contracts node III. It will turn out that this is identical to deriving a condition such that the (positive) direct effect (A) on π_{III}^H exceeds the (negative) indirect effect (B) on π_{III}^H.

<u>(1) The Direct Effect (A)</u>: Substitute (A-14) in (A-13) and rewrite to get,

$$\pi_{III}^H = \{\delta^L[R-\alpha_{III}^L]\}\{\delta^L R_N^H - \phi C_{III}^H\}^{-1} \tag{A-17}$$

Differentiating (A-17) w.r.t. α_{III}^L gives the direct effect (A) on $\overset{\approx}{\pi}{}_{III}^H$,

$$-\partial\overset{\approx}{\pi}{}_{III}^H/\partial\alpha_{III}^L = \delta^L\{\delta^L R_N^H - \phi C_{III}^H\}^{-1} > 0. \tag{A-18}$$

The derivative in (A-18) implies that decreasing the interest for bad borrowers in the contracts III node increases the credit granting probability for good borrowers in that node. The direct effect on $\overset{\approx}{\alpha}{}_I^H$ is to compensate the bank for the losses it sustains in reducing α^L_{III}. The zero expected profit condition implies

$$\partial(\text{bank profit})/\partial\alpha^L_{III} + [\partial\alpha^H_I/\partial\alpha^L_{III}].\ \partial(\text{bank profit})/\partial\alpha^H_I = 0.$$

Define profit as expected profit per good borrower at t=0. Then

$$\partial(\text{bank profit})/\partial\alpha^L_{III} = \bar{\delta}^H\nu\delta^L \text{ and } \partial(\text{bank profit})/\partial\alpha^H_I = \delta^H. \text{ Thus, we get}$$

$$\partial\tilde{\tilde{\alpha}}^H_I/\partial\alpha^L_{III} = -\bar{\delta}^H\nu\delta^L[\delta^H]^{-1}. \tag{A-19}$$

<u>(2) The Indirect Effect (B)</u>: We now examine the first period incentive compatibility effects of the reduction in α^L_{III}. We know from (13) that $U_I(\delta^L|\delta^L) = U_I(\delta^H|\delta^L)$. Note that the variables in (A-16) have no impact on $U_I(\delta^L|\delta^L)$. Hence, the above equality implies

$$-\partial U_I(\delta^H|\delta^L)/\partial\alpha^L_{III} = 0.$$

where

$$U_I(\delta^H|\delta^L) = \delta^L[R-\alpha^H_I] - \bar{\delta}^L C^H_I + \delta^L\{[1-\mu]\delta^H + \mu\delta^L\}\{R-\alpha^H_{II}\}$$
$$- \bar{\delta}^L\delta^L[R-\alpha^L_{III}].$$

Differentiating w.r.t. α^L_{III} gives (constraint on α^H_{II} is binding in the intertemporal solution)

$$-\partial U_I(\delta^H|\delta^L)/\partial\alpha^L_{III} = 0 = \delta^L\{[\partial\tilde{\tilde{\alpha}}^H_I/\partial\alpha^L_{III}] + [\partial\alpha^H_I/\partial\alpha^L_{III}]\}$$
$$- \bar{\delta}^L[\partial C^H_I/\partial\alpha^L_{III}] + \bar{\delta}^L\delta^L. \tag{A-20}$$

where $\partial\tilde{\tilde{\alpha}}^H_I/\partial\alpha^L_{III}$ is given in (A-19) and $\partial\alpha^L_I/\partial\alpha^L_{III}$ and $\partial C^H_I/\partial\alpha^L_{III}$ are such that incentive compatibility is guaranteed and the bank's expected profit stays at zero. The latter requirement implies that

$$\delta^H[\partial\alpha^H_I/\partial\alpha^L_{III}] + \bar{\delta}^H\beta[\partial C^H_I/\partial\alpha^L_{III}] = 0. \tag{A-21}$$

Substituting (A-19) and (A-21) in (A-20) and rearranging produces

$$-\partial C^H_I/\partial\alpha^L_{III} = \{\delta^L[\delta^H\bar{\delta}^L - \nu\delta^L\bar{\delta}^H][\delta^H]^{-1}\}\{[\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H][\delta^H]^{-1}\}^{-1}$$
$$= \delta^L\xi\phi^{-1}, \tag{A-22}$$

where $\xi \equiv [\delta^H\bar{\delta}^L - \nu\delta^L\bar{\delta}^H][\delta^H]^{-1}$.

Substitute (A-22) in (A-21) to get

$$-\partial\alpha_{I}^{H}/\partial\alpha_{III}^{L} = -\{\delta^{L}\bar{\delta}^{H}\beta\xi\}\{\phi\delta^{H}\}^{-1}. \tag{A-23}$$

From (A-22) and (A-23) we see that the incentive compatibility problems which arise from the reduction in α_{III}^{L} are resolved by increasing the first period collateral requirement. The reduction in α_{I}^{H} is to preserve zero expected profit for the bank. From (A-15) we see that

$$-\partial C_{III}^{H}/\partial\alpha_{III}^{L} = -[-\partial C_{I}^{H}/\partial\alpha_{III}^{L}]. \tag{A-24}$$

Combine (A-24) and (A-22) to obtain

$$-\partial C_{III}^{H}/\partial\alpha_{III}^{L} = -\delta^{L}\xi\phi^{-1}. \tag{A-25}$$

The change in C_{III}^{H} has an effect on π_{III}^{H}. From (A-17) we get

$$\partial\pi_{III}^{H}/\partial C_{III}^{H} = \pi_{III}^{H}\phi\{\delta^{L}R_{N}^{H} - \phi C_{III}^{H}\}^{-1}.$$

Combining this with (A-25) we get

$$\begin{aligned} -\partial\pi_{III}^{H}/\partial\alpha_{III}^{L} &= -[\partial\pi_{III}^{H}/\partial C_{III}^{H}]\cdot[\partial C_{III}^{H}/\partial\alpha_{III}^{L}] \\ &= -\pi_{III}^{H}\delta^{L}\xi\{\delta^{L}R_{N}^{H} - \phi C_{III}^{H}\}^{-1} \end{aligned} \tag{A-26}$$

From (A-26) we see that the indirect effect of the reduction in α_{III}^{L} is to reduce the credit granting probability. The sufficiency condition we will derive guarantees that the indirect effect in (A-26) never offsets the (positive) direct effect in (A-18).

With (A-19), (A-23), (A-22), (A-18), (A-26) and (A-24), the relations between the adjustments in (A-16) are determined. We will now derive a sufficiency condition for the adjustments to have a positive impact on the good borrower's utility. This condition is identical to the sufficiency condition for the suboptimality of rationing. Recall that

$$\begin{aligned} U_{I}(\delta^{H}|\delta^{H}) = {} & \delta^{H}[R-\alpha_{I}^{H}] - \bar{\delta}^{H}C_{I}^{H} + [\delta^{H}]^{2}[R - \alpha_{II}^{H}] \\ & + [1-\nu]\pi_{III}^{H}\{\delta^{H}R_{N}^{H} - [1-\beta]\bar{\delta}^{H}C_{III}^{H}\} \\ & - \bar{\delta}^{H}\nu\delta^{L}[R - \alpha_{III}^{L}]. \end{aligned} \tag{A-27}$$

The reduction in α^L_{III} has direct and indirect effects on $U^H_I(\delta^H|\delta^H)$. From (A-27) we get

$$
\begin{aligned}
&- dU_I(\delta^H|\delta^H)/d\alpha^L_{III} \\
&= - \delta^H\{[-\partial\tilde{\tilde{\alpha}}^H_I/\partial\alpha^L_{III}] + [-\partial\alpha^H_I/\partial\alpha^L_{III}]\} - \bar{\delta}^H[-\partial C^H_I/\partial\alpha^L_{III}] \\
&\quad + \bar{\delta}^H[1-\nu][\{-\partial\tilde{\tilde{\pi}}^H_{III}/\partial\alpha^L_{III}\} + \{-\partial\pi^H_{III}/\partial\alpha^L_{III}\}]\{\delta^H R^N_H - [1-\beta]\bar{\delta}^H C^H_{III}\} \\
&\quad - \bar{\delta}^H[1-\nu]\pi^H_{III}[1-\beta]\bar{\delta}^H[-\partial C^H_{III}/\partial\alpha^L_{III}] - \bar{\delta}^H\nu\delta^L[-\partial\alpha^L_{III}/\partial\alpha^L_{III}]
\end{aligned}
$$

Upon substituting (A-18), (A-19), (A-22), (A-23), (A-24) and (A-26) in the above expression, we get

$$
\begin{aligned}
&- dU_I(\delta^H|\delta^H)/d\alpha^L_{III} \\
&= - \delta^H\{\bar{\delta}^H\nu\delta^L[\delta^H]^{-1} - \delta^L\bar{\delta}^H\beta\xi[\phi\delta^H]^{-1}\} - \bar{\delta}^H\delta^L\xi\phi^{-1} \\
&\quad - \bar{\delta}^H[1-\nu]\{\delta^L G_6{}^{-1} - \pi^H_{III}\delta^L\xi G_6{}^{-1}\}\{\delta^H R^H_N - [1-\beta]\bar{\delta}^H C^H_{III}\} \\
&\quad - \bar{\delta}^H[1-\nu]\pi^H_{III}[1-\beta]\bar{\delta}^H\delta^L\xi\phi^{-1} + \bar{\delta}^H\nu\delta^L. \qquad \text{(A-28)}
\end{aligned}
$$

where $G_6 \equiv \delta^L R^H_N - \phi C^H_{III}$.

Unfortunately, it is not possible to evaluate (A-28). Also, the second derivative of $U_I(\delta^H|\delta^H)$ w.r.t. α^L_{III} is ambiguous. Hence, we have to look for a sufficiency condition. We want to show that (A-28) is positive. First, we note that $\delta^L - \pi^H_{III}\delta^L\xi > 0$. Thus, substituting the finding of Lemma 2 in (A-28) will reduce (A-28). That is,

$$
\begin{aligned}
&- dU_I(\delta^H|\delta^H)/d\alpha^L_{III} \\
&\geq \delta^L\bar{\delta}^H\beta\xi\phi^{-1} - \delta^L\bar{\delta}^H\xi\phi^{-1} + \bar{\delta}^H[1-\nu]\delta^L[1-\pi^H_{III}\xi]\delta^H[\delta^L]^{-1} \\
&\quad + [\bar{\delta}^H]^2[1-\nu][1-\beta]\pi^H_{III}\delta^L\xi\phi^{-1}. \qquad \text{(A-29)}
\end{aligned}
$$

Note that the right hand side (RHS) of (A-29) is minimized for $\pi^H_{III} = 1$. To see this, observe that

$$
\begin{aligned}
&\partial[dU_I(\delta^H|\delta^H)/d\alpha^L_{III}]/\partial\pi^H_{III} \\
&= - \bar{\delta}^H[1-\nu]\delta^H\xi + [\bar{\delta}^H]^2[1-\nu][1-\beta]\delta^L\xi\phi^{-1} < 0
\end{aligned}
$$

since $- \delta^H + \bar{\delta}^H[1-\beta]\delta^L\phi^{-1} < 0$.

Substituting $\pi^H_{III} = 1$ we get

$$\begin{aligned} &- dU_I(\delta^H|\delta^H)/d\alpha^L_{III} \\ &\geq -[1-\beta]\delta^L\bar{\delta}^H\xi\phi^{-1} + \bar{\delta}^H[1-\nu]\delta^L[1-\xi]\delta^H[\delta^L]^{-1} \\ &\quad + [\bar{\delta}^H]^2[1-\nu][1-\beta]\delta^L\xi\phi^{-1}. \end{aligned} \tag{A-30}$$

The RHS of (A-30) is non-negative if

$$-[1-\beta]\xi\phi^{-1} + [1-\nu][1-\xi]\delta^H[\delta^L]^{-1} + \bar{\delta}^H[1-\nu][1-\beta]\xi\phi^{-1} \geq 0.$$

This implies that we need

$$\phi[1-\nu][1-\xi]\delta^H[\delta^L]^{-1} \geq [1-\beta]\xi[1-\{1-\nu\}\bar{\delta}^H].$$

Substitute the expressions for ξ and ϕ in the inequality above and note that $1 - [1-\nu]\bar{\delta}^H = \delta^H - \nu\bar{\delta}^H$, to obtain

$$\begin{aligned} &\{\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H\}\{\delta^H\}^{-1}\{1-\nu\}\delta^H\{\delta^L\}^{-1}\{\delta^L[\delta^H + \nu\bar{\delta}^H][\delta^H]^{-1}\} \\ &\geq [1-\beta]\{\delta^H\bar{\delta}^L - \nu\delta^L\bar{\delta}^H\}\{\delta^H\}^{-1}\{\delta^H - \nu\bar{\delta}^H\}. \end{aligned}$$

which implies

$$[1-\nu][\delta^H]^{-1}[\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H] \geq [1-\beta][\delta^H\bar{\delta}^L - \nu\delta^L\bar{\delta}^H][\delta^H]^{-1}.$$

which holds if

$$\nu \leq \beta. \qquad \text{Q.E.D.}$$

<u>PROOF OF THEOREM 4</u>: Apply the Simplex algorithm. Take into account only the constraints (19), (21), (24) and (25). First, we add slack variables to these constraints. Substituting (26), we obtain

$$- \delta^H\alpha^H_I - \bar{\delta}^H\beta C^H_I - [\delta^H]^2\alpha^H_{II} - \bar{\delta}^H[1-\nu]\delta^H\alpha^H_{III} - \bar{\delta}^H\nu\delta^L\alpha^L_{III}$$
$$+ S_1 = -2r + [\bar{\delta}^H]^2[1-\nu]\theta\beta\delta r[\phi\delta^H]^{-1}. \tag{19)'$$

$$\delta^H\alpha^H_{III} \qquad + S_2 = r - \bar{\delta}^H\theta\beta r\delta[\phi\delta^H]^{-1}. \tag{21)'$$

$$-\delta^L\alpha^H_{III} + \delta^L\alpha^L_{III} \qquad - S_3 = \bar{\delta}^L\theta r\delta[\phi\delta^H]^{-1}. \tag{24)'$$

$$-\delta^L\alpha^H_I - \bar{\delta}^L C^H_I - \delta^L[\delta^H - \delta\mu]\alpha^H_{II} - \bar{\delta}^L\delta^L\alpha^L_{III} - S_4 = -2r$$

In the tableaus I through V, we apply the Simplex algorithm. Tableau V is the

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TABLEAU I:

Basis	C_j	α_I^H	C_I^H	α_{II}^H	α_{III}^H	α_{III}^L	S_1	S_2	S_3	S_4	
		$-\delta^H$	$-(1-\delta^H)$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)\nu\delta^L$	0	0	0	0	
S_1	0	$-\delta^H$	$-(1-\delta^H)\beta$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)\nu\delta^L$	1	0	0	0	$-2r+(1-\delta^H)^2(1-\nu)\dfrac{\theta\beta(\delta^H-\delta^L)r}{\delta^H\phi}$
S_2	0	0	0	0	δ^H	0	0	1	0	0	$r-(1-\delta^H)\dfrac{\theta\beta(\delta^H-\delta^L)r}{\delta^H\phi}$
S_3	0	0	0	0	$-\delta^L$	δ^L	0	0	1	0	$(1-\delta^L)\dfrac{\theta(\delta^H-\delta^L)r}{\delta^H\phi}$
S_4	0	$-\delta^L$	$-(1-\delta^L)$	$-\delta^L[\delta^H-\mu(\delta^H-\delta^L)]$	0	$-(1-\delta^L)\delta^L$	0	0	0	1	$-2r$
	Z_j	0	0	0	0	0	0	0	0	0	
	C_j-Z_j	$-\delta^H$	$-(1-\delta^H)$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)\nu\delta^L$	0	0	0	0	

(1) Row 1 $\rightarrow$ Add $-\delta^H/\delta^L$ (Row 4)

(2) Row 4 $\rightarrow$ Multiply by $-1/\delta^L$ => α_I^H in Basis

TABLEAU II:

Basis	C_j	a_I^H	c_I^H	a_{II}^H	a_{III}^H	a_{III}^L	S_1	S_2	S_3	S_4	
		$-\delta^H$	$-(1-\delta^H)$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)\nu\delta^L$	0	0	0	0	
S_1	0	0	$\phi\delta^H/\delta^L$	$-\mu\delta^H(\delta^H-\delta^L)$	$(1-\delta^H)(1-\nu)\delta^H$	$\delta^H\xi$	1	0	0	$-\delta^H/\delta^L$	$-\frac{2(\delta^H-\delta^L)r}{\delta^L}+(1-\delta^H)^2(1-\nu)\frac{\theta B(\delta^H-\delta^L)r}{\delta^H\phi}$
S_2	0	0	0	0	δ^H	0	0	1	0	0	$r-(1-\delta^H)\frac{\theta B(\delta^H-\delta^L)r}{\delta^H\phi}$
S_3	0	0	0	0	$-\delta^L$	δ^L	0	0	1	0	$(1-\delta^L)-\frac{\theta(\delta^H-\delta^L)r}{\delta^H\phi}$
a_I^H	$-\delta^H$	1	$(1-\delta^L)/\delta^L$	$\delta^H-\mu(\delta^H-\delta^L)$	0	$(1-\delta^L)$	0	0	0	$-1/\delta^L$	$2r/\delta^L$
	Z_j	$-\delta^H$	$-(1-\delta^L)\delta^H/\delta^L$	$-\delta^H[\delta^H-\mu(\delta^H-\delta^L)]$	0	$-\delta^H(1-\delta^L)$	0	0	0	δ^H/δ^L	
	C_j-Z_j	0	$\frac{\delta^H-\delta^L}{\delta^L}$	$-\mu\delta^H(\delta^H-\delta^L)$	$-(1-\delta^H)(1-\nu)\delta^H$	$\xi\delta^H$	0	0	0	$-\delta^H/\delta^L$	

(1) Row 4 → Add $-\frac{\delta^L}{\delta^H\phi}\cdot\frac{1-\delta^L}{\delta^L}$ (Row 1)

(2) Row 1 → Multiply by $\delta^L/\delta^H\phi$

(3) C_j-Z_j Row → Add $-\frac{\delta^L}{\delta^H\phi}\cdot\frac{\delta^H-\delta^L}{\delta^H}\cdot$ (Row 1)

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TABLEAU III:

Basis	c_j	a_I^H	c_I^H	a_{II}^H	a_{III}^H	a_{III}^L	S_2	S_3	S_4	S_5	
		$-\delta^H$	$-(1-\delta^H)$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)\nu\delta^L$	0	0	0	0	
c_I^H	$-(1-\delta^H)$	0	1	$-\mu\delta^L(\delta^H-\delta^L)/\phi$	$-\delta^L(1-\delta^H)(1-\nu)/\phi$	$\delta^L\xi/\phi$	$\frac{\delta^L}{\phi\delta^H}$	0	0	$-1/\phi$	$-\frac{2(\delta^H-\delta^L)r}{\phi\delta^H}+(1-\delta^H)^2(1-\nu)\frac{\theta\beta\delta^L(\delta^H-\delta^L)r}{\delta^H\delta^H\phi\phi}$
S_2	0	0	0	0	δ^H	0	0	1	0	0	$r-(1-\delta^H)\frac{\theta\beta(\delta^H-\delta^L)r}{\delta^H\phi}$
S_3	0	0	0	0	$-\delta^L$	δ^L	0	0	1	0	$(1-\delta^L)\ \frac{\theta(\delta^H-\delta^L)r}{\delta^H\phi}$
a_I^H	$-\delta^H$	1	0	$\delta^H-\mu(\delta^H-\delta^L)$ $+\mu(1-\delta^L)\frac{(\delta^H-\delta^L)}{\phi}$	$(1-\delta^L)(1-\delta^H)(1-\nu)/\phi$	$(1-\delta^L)(1-\xi/\phi)$	$-\frac{(1-\delta^L)}{\phi\delta^H}$	0	0	$\frac{(1-\delta^L)}{\phi\delta^L}-1/\delta^L$	$2r/\delta^L-\frac{2(1-\delta^L)(\delta^H-\delta^L)r}{\phi\delta^H\delta^L}$ $-(1-\delta^H)^2(1-\nu)(1-\delta^L)\frac{\theta\beta(\delta^H-\delta^L)r}{\delta^H\delta^H\phi\phi}$
	z_j					$\xi\delta^H-(\delta^H-\delta^L)\xi/\phi$	$-\frac{(\delta^H-\delta^L)}{\phi\delta^H}$	0	0	$-\delta^H/\delta^L$ $+\frac{(\delta^H-\delta^L)}{\delta^L\phi}$	
	c_j-z_j	0	0	$-\mu\delta^H(\delta^H-\delta^L)$ $+\mu(\delta^H-\delta^L)^2/\phi$ ≤ 0	$-(1-\delta^H)(1-\nu)\delta^H$ $+(1-\delta^H)(\delta^H-\delta^L)(1-\nu)/\phi$						

(1) Row 1 => Add $-(\frac{1}{\delta^L})(\delta^L\xi/\phi)\cdot$ (Row 3)

(2) Row 4 => Add $-(\frac{1}{\delta^L})\frac{\delta^L(1-\delta^H)(1-\delta^L)(\nu-\beta)}{\delta^H\phi}$ (Row 3)

(3) Row 3 => Multiply by $1/\delta^L$

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TABLEAU IV:

Basis	c_j	a_I^H	c_I^H	a_{II}^H	a_{III}^H	a_{III}^L	s_1	s_2	s_3	s_4	
		$-\delta^H$	$-(1-\delta^H)$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$(1-\delta^H)\nu\delta^L$	0	0	0	0	
c_I^H	$-(1-\delta^H)$	0	1	$-\mu\delta^L(\delta^H-\delta^L)/\phi$	$-\frac{\delta^L(\delta^H-\delta^L)}{\phi\delta^H}[\delta^H+\nu(1-\delta^H)]$	0	$\delta^L/\phi\delta^H$	0	$-\xi/\phi$	$1/\phi$	$\frac{2(\delta^H-\delta^L)r}{\phi\delta^H} - \frac{(1-\delta^L)\theta(\delta^H-\delta^L)\xi r}{\delta^H\phi\phi}$ $+ (1-\delta^H)^2(1-\nu)\frac{\theta\beta\delta^L(\delta^H-\delta^L)r}{\delta^H\delta^H\phi\phi}$
s_2	0	0	0	0	δ^H	0	0	1	0	0	$r-(1-\delta^H)\frac{\theta\beta(\delta^H-\delta^L)r}{\delta^H\phi}$
a_{III}^L	$-(1-\delta^H)\nu\delta^L$	0	0	0	-1	1	0	0	$1/\delta^L$	0	$\frac{1-\delta^L}{\delta^L} + \frac{\theta(\delta^H-\delta^L)r}{\delta^H\phi}$
a_I^H	$-\delta^H$	1	0	$\delta^H-\mu(\delta^H-\delta^L)$ $+\mu(1-\delta^L)(\delta^H-\delta^L)/\phi$	$-\frac{(1-\delta^H)(1-\delta^L)}{\delta^H\phi}[\delta^H-\delta^L\beta]$ $-\frac{(1-\delta^H)(1-\delta^L)}{\delta^H\phi}(\delta^H-\delta^L)\nu$	0	$-(1-\delta^L)/\phi\delta^H$	0	$-\frac{(1-\delta^H)(1-\delta^L)(\nu-\beta)}{\delta^H\phi}$	$\frac{(1-\delta^L)}{\phi\delta^L}$ $1/\delta^L$	$2r/\delta^L - \frac{2(1-\delta^L)(\delta^H-\delta^L)r}{\phi\delta^H\delta^L}$ $-(1-\delta^H)^2(1-\nu)(1-\delta^L)\frac{\theta\beta(\delta^H-\delta^L)r}{\delta^H\delta^H\phi\phi}$ $-\frac{(1-\delta^H)(1-\delta^L)^2(\nu-\beta)\theta(\delta^H-\delta^L)r}{\delta^H\delta^H\phi\phi}$
	z_j c_j-z_j	0	0	$-\mu\delta^H(\delta^H-\delta^L)$ $+\mu(\delta^H-\delta^L)^2/\phi$ ≤ 0	$-(1-\delta^H)(1-\nu)\delta^H$ $+(1-\delta^H)(\delta^H-\delta^L)(1-\nu)/\phi$ $+\frac{\xi\phi\delta^H-(\delta^H-\delta^L)\xi}{\phi}$ $\frac{[\xi-(1-\delta^H)(1-\nu)][\phi\delta^H-(\delta^H-\delta^L)]}{\phi} \geq 0$	0	$-(\delta^H-\delta^L)/\phi\delta^H$ < 0	0	$-\frac{\xi\phi\delta^H-(\delta^H-\delta^L)\xi}{\delta^L\phi}$ < 0	δ^H/δ^L $+\frac{(\delta^H-\delta^L)}{\delta^L\phi}$ ≤ 0	

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TABLEAU V:

Basis	c_j	e_I^M	c_I^M	e_{II}^M	e_{III}^M	e_{III}^L	s_1	s_2	s_3	s_4	
		$-\delta^M$	$-(1-\delta^M)$	$-\delta^M\delta^M$	$-(1-\delta^M)(1-v)\delta^M$	$(1-\delta^M)v\delta^L$	0	0	0	0	
c_I^M	$-(1-\delta^M)$	0	1	$-\mu(\delta^M-\delta^L)/\phi$	0	0	$\frac{\delta^L}{\phi\delta^M}$	$\frac{\delta^L(\delta^M-\delta^L)}{\phi\delta^M\delta^M}$	$-1/\phi$	$1/\phi$	$\frac{2(\delta^M-\delta^L)r}{\phi\delta^M} - \frac{(1-\delta^L)\theta(\delta^M-\delta^L)[r}{\delta^M\phi\phi} + (1-\delta^M)^2(1-v)\frac{\theta\theta\delta^L(\delta^M-\delta^L)r}{\delta^M\delta^M\phi\phi}$ $-\frac{\delta^L(\delta^M-\delta^L)}{\phi\delta^M\delta^M}[\delta^M+v(1-\delta^M)][r-\frac{(1-\delta^M)\theta\theta(\delta^M-\delta^L)r}{\phi\delta^M}]$
e_{III}^M	$-(1-\delta^M)(1-v)\delta^M$	0	0	0	1	0	0				$r/\delta^M - \frac{1-\delta^M}{\delta^M}\cdot\frac{\theta\theta(\delta^M-\delta^L)r}{\phi\delta^M}$
e_{III}^L	$-(1-\delta^M)v\delta^L$	0	0	0	0	1	0				$\frac{1-\delta^L}{\delta^L}\cdot\frac{\theta(\delta^M-\delta^L)r}{\delta^M\phi} + r/\delta^M - \frac{1-\delta^M}{\delta^M}\frac{\theta\theta(\delta^M-\delta^L)r}{\phi\delta^M}$
e_I^M	$-\delta^M$	1	0	$\delta^M-\mu(\delta^M-\delta^L)$ $+\mu(1-\delta^L)(\delta^M-\delta^L)/\phi$	0	0	$\frac{(1-\delta^L)}{\phi\delta^M}$	$[\frac{(1-\delta^M)(1-\delta^L)}{\phi\delta^M\delta^M}]$ $[(\delta^M-\delta^L)v-\delta^M+\theta\delta^L]$	$\frac{(1-\delta^M)(1-\delta^L)(v-\theta)}{\phi\delta^M}$	$\frac{1-\delta^L}{\phi\delta^L}-1/\delta^L$	$2r/\delta^L - \frac{2(1-\delta^L)(\delta^M-\delta^L)r}{\phi\delta^M\delta^L}$ $-(1-\delta^M)^2(1-v)(1-\delta^L)\frac{\theta\theta(\delta^M-\delta^L)r}{\phi\phi\delta^M\delta^M}$ $-\frac{(1-\delta^M)(1-\delta^L)^2(v-\theta)\theta(\delta^M-\delta^L)r}{\delta^M\delta^M\phi\phi}$ $-\frac{(1-\delta^M)(1-\delta^L)}{\phi\delta^M\delta^M}[\delta^M-\delta^L\theta-(\delta^M-\delta^L)v]\cdot$ $[r-(1-\delta^M)\frac{\theta\theta(\delta^M-\delta^L)r}{\phi\delta^M}]$
	z_j										
	c_j-z_j	0	0	$\mu\delta^M(\delta^M-\delta^L)$ $+\mu(\delta^M-\delta^L)^2/\phi$ ≤ 0	0	0	$\frac{\delta^M-\delta^L}{\phi\delta^M}$ < 0	$-\frac{[(1-\delta^M)(1-v)][\phi\delta^M-(\delta^M-\delta^L)]}{\phi\delta^M}$ ≤ 0	$\frac{[\phi\delta^M-(\delta^M-\delta^L)]}{\phi\delta^L}$ < 0	δ^M/δ^L $+\frac{\delta^M-\delta^L}{\phi\delta^L}$ ≤ 0	

This is the final Tableau because (1) all $C_j - Z_j \leq 0$,
(2) all values in last column positive.

end-tableau.

[INSERT TABLEAUS ABOUT HERE]

Recalling that ξ was defined immediately preceding Lemma 1 and in the proof of Theorem 3, the solution indicates

$$\tilde{C}_I^H = \delta r[\Phi\delta^H]^{-1}\{2 - \bar{\delta}^L\Theta\xi\Phi^{-1} + [\bar{\delta}^H]^2[1-\nu]\delta^L\Theta\beta[\Phi\delta^H]^{-1} - G_7 + G_8\}$$

where $G_7 \equiv \delta^L[\delta^H + \nu\bar{\delta}^H][\delta^H]^{-1}$

$$G_8 \equiv \delta^L[\delta^H + \nu\bar{\delta}^H]\delta\bar{\delta}^H\Theta\beta[\Phi\{\delta^H\}^2]^{-1}.$$

After considerable algebra, the above expression simplifies to

$$\tilde{C}_I^H = \delta r[1 - \{1-\Theta\}\xi][\Phi\delta^H]^{-1}.$$

The solution also indicates that

$$\tilde{\alpha}_I^H = 2r[\delta^L]^{-1} - 2\bar{\delta}^L[\delta^L]^{-1}\delta[\Phi\delta^H]^{-1} - G_9 - G_{10} - G_{11}$$

where $G_9 \equiv [\bar{\delta}^H]^2[1-\nu]\bar{\delta}^L\Theta\beta\delta r[\Phi^2\{\delta^H\}^2]^{-1}$

$$G_{10} \equiv \bar{\delta}^H[\bar{\delta}^L]^2[\nu-\beta]\Theta\delta r[\{\delta^H\}^2\Phi^2]^{-1}$$

$$G_{11} \equiv \bar{\delta}^H\bar{\delta}^L[\Phi\{\delta^H\}^2]^{-1}[\delta^H - \delta^L\beta - \delta\nu][r - \bar{\delta}^H\Theta\beta\delta\{\Phi\delta^H\}^{-1}]$$

Again, after considerable algebra, we can simplify this expression to

$$\tilde{\alpha}_I^H = [1-\delta^H]r[\delta^H]^{-1} - \bar{\delta}^H\delta r\{\beta[1-\{1-\Theta\}\xi]\}\{\Phi[\delta^H]^2\}^{-1}$$
$$- \bar{\delta}^H\delta r[1-\Theta]\nu\Phi[\{\delta^H\}^2\Phi]^{-1}$$

Substituting the solution for $\tilde{C}_I^H$ in the above expression, we get

$$\tilde{\alpha}_I^H = [1-\delta^H]r[\delta^H]^{-1} - \bar{\delta}^H[\delta^H]^{-1}\beta\tilde{C}_I^H - \bar{\delta}^H\delta r[1-\Theta]\nu\Phi\{[\delta^H]^2\Phi\}^{-1}.$$

The solution also indicates that

$$\tilde{\alpha}_{III}^H = r[\delta^H]^{-1} - \Theta\beta\delta r\bar{\delta}^H[\Phi\{\delta^H\}^2]^{-1}.$$

Using (26) gives us

$$\tilde{\alpha}_{III}^H = r[\delta^H]^{-1} - \bar{\delta}^H\beta\tilde{C}_{III}^H[\delta^H]^{-1}.$$

Furthermore,

$$\tilde{\alpha}_{III}^L = r[\delta^H]^{-1} + [\bar{\delta}^L\{\delta^L\}^{-1} - \bar{\delta}^H\beta\{\delta^H\}^{-1}]\Theta\delta r[\Phi\delta^H]^{-1}$$
$$= r[\delta^H]^{-1} + \Phi\Theta\delta r[\Phi\delta^H\delta^L]^{-1}.$$

which can be written as

$$\tilde{\alpha}^L_{III} = r[\delta^L]^{-1} - \Phi[1-\Theta]\delta r[\Phi\delta^L\delta^H]^{-1}.$$

Note also that α^H_{III} is not a basis variable. Thus, the Simplex algorithm implies that

$$\tilde{\alpha}^H_{II} = 0.$$

Finally, by (26) and our collateral assumption, we obtain

$$\tilde{C}^H_{III} = \Theta\delta r[\Phi\delta^H]^{-1}.$$

So, we have determined the intertemporal contract that a good borrower gets at t=0. As argued earlier, bad borrowers obtain a contract that yields an average expected utility per period equal to the first best for them. Thus, their total expected interest cost is 2r. Generally, there is no unique solution. However, it is easy to see that the solution $\{\tilde{\alpha}^L_I, \tilde{\alpha}^H_{IV}, \tilde{\alpha}^L_{IV}, \tilde{\alpha}^L_V\}$ as stated in the theorem implies an expected interest cost of 2r, and cannot be broken by spot market competitors at t=1. Q.E.D.

<u>PROOF OF LEMMA 3</u>: The proof is by contradiction. Note that $\tilde{C}^H_{III} = \Theta\delta r[\Phi\delta^H]^{-1}$. If it is optimal <u>not</u> to use all available collateral, then reducing Θ should increase the welfare of a good borrower at t=0. Now

$$\begin{aligned}
&-\partial U_I(\delta^H|\delta^H)/\partial\Theta \\
&= \delta^H[\partial\tilde{\alpha}^H_I/\partial\Theta] + \bar{\delta}^H[\partial\tilde{C}^H_I/\partial\Theta] + \bar{\delta}^H[1-\nu][\delta^H\{\partial\tilde{\alpha}^H_{III}/\partial\Theta\} + \bar{\delta}^H\{\partial\tilde{C}^H_{III}/\partial\Theta\}] \\
&\quad + \bar{\delta}^H\nu\delta^L[\partial\alpha^L_{III}/\partial\Theta].
\end{aligned}$$

(For the definition of $U_I(\delta^H|\delta^H)$, see (3). The parametric expressions for the various instruments are defined in Theorem 4). We can thus write

$$\begin{aligned}
&-\partial U_I(\delta^H|\delta^H)/\partial\Theta \\
&= -\bar{\delta}^H\delta\beta r[\Phi\delta^H]^{-1} - \bar{\delta}^H\delta r\nu[\delta^H]^{-1} - \bar{\delta}^H\delta r\xi[\Phi\delta^H]^{-1} \\
&\quad -\bar{\delta}^H[1-\nu]\{\bar{\delta}^H\beta\delta r[\Phi\delta^H]^{-1} - \bar{\delta}^H\delta r[\Phi\delta^H]^{-1}\} + \bar{\delta}^H\nu\delta r[\delta^H]^{-1}.
\end{aligned}$$

Thus,

$$-\partial U_I(\delta^H|\delta^H)/\partial\theta$$
$$= -\bar{\delta}^H[1-\beta]\delta r[\xi - \bar{\delta}^H\{1-\nu\}][\phi\delta^H]^{-1} < 0, \text{ since } \xi - \bar{\delta}^H[1-\nu] < 0.$$

This means reducing θ worsens the lot of the good borrower. Q.E.D.

<u>PROOF OF LEMMA 4</u>: From Theorem 2 we know that

$$\hat{C}_I^H = \delta r[\phi\delta^H]^{-1}[1 + \mu\delta^L - \{1-\beta\}\bar{\delta}^H\delta^L\{1-\mu\}\phi^{-1}]. \tag{A-31}$$

and from Theorem 4 we know that

$$\tilde{C}_I^H = \delta r[\phi\delta^H]^{-1}[1+\{1-\theta\}\xi]. \tag{A-32}$$

In both cases we used the assumption that

$$C_I^H \le W_0 < C_I^H + C_{III}^H(\text{opt}), \text{ where } C_{III}^H(\text{opt}) = \delta r[\phi\delta^H]^{-1}.$$

In (A-31), this assumption becomes

$$\delta r[\phi\delta^H]^{-1}[1 - \mu\delta^L - G_{12}] \le W_0 < \delta r[\phi\delta^H]^{-1}[2 + \mu\delta^L - G_{12}], \tag{A-33}$$

where $G_{12} \equiv \{1-\beta\}\bar{\delta}^H\delta^L\{1-\mu\}\phi^{-1}$. In (A-32), this assumption becomes (note $\theta \in [0, 1)$)

$$\delta r[1 + \xi][\phi\delta^H]^{-1} \le W_0 < 2\delta r[\phi\delta^H]^{-1}. \tag{A-34}$$

We look for levels of collateral, W_0, which satisfy (A-33) and (A-34) simultaneously. It follows directly that the conditions in (27) guarantee this. (Note that, because $\mu\delta^L - G_{12} < 1$, the lower bound on W_0 in (A-33) does not exceed the upper bound in (A-34)). Q.E.D.

<u>PROOF OF LEMMA 5</u>: The variables in contracts node III in the single period contract solution are

$$\hat{\alpha}_{III}^H = r[\delta^H]^{-1} - \bar{\delta}^H\beta\hat{C}_{III}^H[\delta^H]^{-1}, \quad \hat{C}_{III}^H = W,$$
$$\hat{\pi}_{III}^H = \delta^L[R-\hat{\alpha}_{III}^L]\{\delta^L[R-\hat{\alpha}_{III}^H] - \bar{\delta}^L W\}^{-1}; \tag{A-35}$$

$$\hat{\alpha}^L_{III} = r[\delta^L]^{-1},\ \hat{C}^L_{III} = 0,\ \hat{\pi}^L_{III} = 1. \tag{A-36}$$

Instead, the bank could have offered a pooling contract, taking into account the proportions ν and $1 - \nu$ of bad and good borrowers, respectively. The zero expected profit condition for the bank dictates that $\bar{\alpha}_{III}$ should satisfy

$$\nu\delta^L[\bar{\alpha}_{III}] + [1 - \nu]\delta^H[\bar{\alpha}_{III}] = r,$$

which implies

$$\bar{\alpha}_{III} = r\{\nu\delta^L + [1-\nu]\delta^H\}^{-1}. \tag{A-37}$$

Existence of equilibrium is guaranteed if good borrowers prefer the contract in (A-35) to the pooling contract in (A-37). Thus, we need

$$U_{III}(\delta^H|\delta^H)\Big|_{\{\hat{\alpha}^H_{III},\ \hat{C}^H_{III},\ \hat{\pi}^H_{III}\}} \geq U_{III}(\delta^H|\delta^H)\Big|_{\bar{\alpha}_{III}}$$

This means we need

$$\hat{\pi}^H_{III}[\delta^H\{R - \hat{\alpha}^H_{III}\} - \bar{\delta}^H\hat{C}^H_{III}] \geq \delta^H[R - \bar{\alpha}_{III}].$$

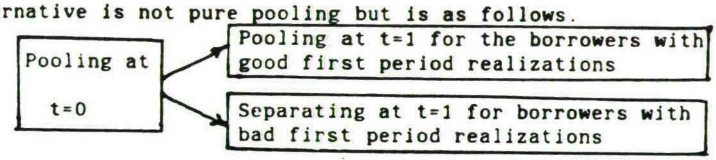
Upon substituting (A-35) and (A-37), the above inequality becomes

$$\{\delta^H R^H_N - \bar{\delta}^H[1-\beta]W\}\{\delta^L R^H_N - \phi W\}^{-1} \geq \{\delta^H[R-r\{\nu\delta^L + [1-\nu]\delta^H\}^{-1}]\}[\delta^L R^L_N]^{-1}. \tag{A-38}$$

For the purpose of this proof we can make the existence restriction (A-38) less restrictive by substituting $W = C_{III}(\text{opt})$. This gives (after rearranging)

$$\bar{\delta}^H[1-\beta][\phi\delta^H]^{-1} \leq \nu[\{1-\nu\}\delta^H + \nu\delta^L]^{-1}. \tag{A-39}$$

This is the existence restriction for an optimally collateralized single period separating contract. It is identical to the condition in footnote 12 in B-T (1987). Recall that (A-39) is less restrictive then (A-38) which is the 'real' existence restriction for the separating contracts III. Hence, if we prove that the existence restriction for separating contracts in the second period pool of borrowers with bad first period realizations is less restrictive than (A-39), then one may conclude that it is also less restrictive than (A-38). The intertemporal pooling contract restriction, to be presented in Theorem 5, indicates that the second period contracts to borrowers with bad first period realizations will be as bad as feasibility permits. This means that these contracts will be the same as spot market contracts. We will now derive the existence restriction for the optimality of separating second period contracts in the pool of borrowers with bad first period realizations. So, the pooling alternative is not pure pooling but is as follows.



It will be apparent later that, except for the original separating solution, no other (pooling/separating) alternatives exist. Since this will cause no confusion, we will refer to the mixed pooling/separating alternative as the pooling contract alternative. The separating contracts are optimally collateralized (notice that no collateral is lost in the pooling contract at t=0). Parametrically, the contracts are identical to the single period contract IV solution presented earlier. The optimal contracts are ('-''s indicate optimal values in the pooling contract solution),

$$\bar{\alpha}_B^H = r[\delta^H]^{-1} - \bar{\delta}^H \beta \bar{C}_B^H [\delta^H]^{-1}, \quad \bar{C}_B^H = \delta r[\phi\delta^H]^{-1}, \quad \bar{\pi}_B^H = 1;$$
$$\alpha_B^L = r[\delta^L]^{-1}, \quad \bar{C}_B^L = 0, \quad \bar{\pi}_B^L = 1. \qquad \text{(A-40)}$$

From Figure 1, we see that the pool of borrowers with bad first period realizations has a fraction Ω of borrowers that are bad in the second period (the rest are good), where

$$\Omega \equiv \{[1-\gamma]\bar{\delta}^H \nu + \gamma\bar{\delta}^L\}\{[1-\gamma]\bar{\delta}^H + \gamma\bar{\delta}^L\}^{-1} \qquad \text{(A-41)}$$

In a manner analogous to (A-37), we can now design a pooling contract based on the proportions Ω and $1 - \Omega$. The pooling interest factor is

$$\bar{\alpha}_2^B = r\{\Omega\delta^L + [1-\Omega]\delta^H\}^{-1}. \qquad \text{(A-42)}$$

Again, existence is guaranteed if

$$U_B(\delta^H)\Big|_{(\bar{\alpha}_B^H,\ \bar{c}_B^H,\ \bar{\pi}_B^H)} \geq U_B(\delta^H)\Big|_{\bar{\alpha}_2^B}. \qquad \text{(A-43)}$$

where $U_B(\delta^H)$ is the expected second period utility of a good borrower in the pool of borrowers with bad first period realizations. That is,

$$U_B(\delta^H) = \pi_B^H\{\delta^H[R - \alpha_B^H] - \bar{\delta}^H c_B^H\}.$$

Upon substituting (A-40) and (A-42) in (A-43), we obtain

$$\bar{\delta}^H[1-\beta][\phi\delta^H]^{-1} \leq \Omega\ \{[1-\Omega]\delta^H - \Omega\delta^L\}^{-1}. \qquad \text{(A-44)}$$

See that (A-44) is similar to (A-39). We will now prove that (A-44) is less restrictive than (A-39). The RHS of (A-39) is strictly increasing in ν, whereas that of (A-44) is strictly increasing in Ω. Hence, it is sufficient to show that $\Omega > \nu$, or equivalently, that

$$\{[1-\gamma]\bar{\delta}^H\nu + \gamma\bar{\delta}^L\}\{[1-\gamma]\bar{\delta}^H + \gamma\bar{\delta}^L\}^{-1} > \nu.$$

But this is certainly true. Q.E.D.

<u>PROOF OF THEOREM 5</u>: Apply the Simplex algorithm. Take into account only the constraints (29), (30), (32), (33) and (35). First, we add slack variables to these constraints to obtain

$$-[\{1-\gamma\}\delta^H + \gamma\delta^L]\alpha_1 - G_{13}\alpha_2^G - G_{14}\delta^H\alpha_B^H - G_{14}\bar{\delta}^H\beta c_B^H - G_{15}\delta^L\alpha_B^L + S_1 = -2r \qquad (29)'$$

$$\delta^L\alpha_1 + \delta^L[\{1-\mu\}\delta^H + \mu\delta^L]\alpha_2^G + \bar{\delta}^L\delta^L\alpha_B^L + S_2 = 2r \qquad (30)'$$

$$\delta^H\alpha_B^H + \bar{\delta}^H\beta c_B^H + S_3 = r \qquad (32)'$$

$$-\delta^L\alpha_B^H - \bar{\delta}^L c_B^H + \delta^L\alpha_B^L + S_4 = 0 \qquad (35)'$$

$$\delta^L\alpha_B^L + S_5 = r \qquad (33)'.$$

where $G_{13} \equiv [1-\gamma][\delta^H]^2 + \gamma\delta^L[1-\mu]\delta^H + \gamma\delta^L\mu\delta^L$

$$G_{14} \equiv [1-\gamma]\bar{\delta}^H[1-\nu]$$

App-19a

TABLEAU 1:

BASIS	C_j	α_1	α_2^G	α_B^H	C_B^H	α_B^L	S_1	S_2	S_3	S_4	S_5	
		$-\delta^H$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)^2(1-\nu)$	$-(1-\delta^H)\nu\delta^L$						
S_1	0	$-[(1-\gamma)\delta^H+\gamma\delta^L]$	$-(1-\gamma)\delta^H\delta^H-\gamma\delta^L(1-\mu)\delta^H$ $-\gamma\delta^L\mu\delta^L$	$-(1-\gamma)(1-\delta^H)(1-\nu)\delta^H$	$-(1-\gamma)(1-\delta^H)^2(1-\nu)\beta$	$-[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L$	1	0	0	0	0	$-2r$
S_2	0	δ^L	$\delta^L[(1-\mu)\delta^H+\mu\delta^L]$	0	0	$(1-\delta^L)\delta^L$	0	1	0	0	0	$2r$
S_3	0	0	0	δ^H	$(1-\delta^H)\beta$	0	0	0	1	0	0	r
S_4	0	0	0	$-\delta^L$	$-(1-\delta^L)$	δ^L	0	0	0	1	0	0
S_5	0	0	0	0	0	δ^L	0	0	0	0	1	r
	Z_j	0	0	0	0	0	0	0	0	0	0	
	C_j-Z_j	$-\delta^H$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)^2(1-\nu)$	$-(1-\delta^H)\nu\delta^L$	0	0	0	0	0	

APP-19b

TABLEAU 2:

		a_1	a_2^G	a_B^H	C_B^H	a_B^L	S_1	S_2	S_3	S_4	S_5	
BASIS	C_j	$-\delta^H$	$-\delta^H\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)^2(1-\nu)$	$-(1-\delta^H)\nu\delta^L$						
a_1	$-\delta^H$	1	$\frac{[(1-\gamma)\delta^H\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{(1-\gamma)(1-\delta^H)(1-\nu)\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{(1-\gamma)(1-\delta^H)^2(1-\nu)B}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{-1}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	0	0	0	$\frac{2r}{(1-\gamma)\delta^H+\gamma\delta^L}$
S_2	0	0	$\frac{\delta^L[(1-\mu)\delta^H+\mu\delta^L]-\delta^L[(1-\gamma)\delta^H\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-\frac{\delta^L(1-\gamma)(1-\delta^H)(1-\nu)\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-\frac{\delta^L(1-\gamma)(1-\delta^H)^2(1-\nu)B}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{(1-\delta^L)\delta^L-[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	1	0	0	0	$\frac{2r-2\delta^L r}{(1-\gamma)\delta^H+\gamma\delta^L}$
S_3	0	0	0	δ^H	$(1-\delta^H)B$	0	0	0	1	0	0	r
S_4	0	0	0	$-\delta^L$	$-(1-\delta^L)$	δ^L	0	0	0	1	0	0
S_5	0	0	0	0	0	δ^L	0	0	0	0	1	r
	Z_j											
	C_j-Z_j	0	$-\delta^H\delta^H+\frac{\delta^H[(1-\gamma)\delta^H\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-(1-\delta^H)(1-\nu)\delta^H+\frac{\delta^H(1-\gamma)(1-\delta^H)^2(1-\nu)\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-(1-\delta^H)^2(1-\nu)+\frac{\delta^H(1-\gamma)(1-\delta^H)^2(1-\nu)B}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-(1-\delta^H)\nu\delta^L+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$ > 0	$\frac{-\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	0	0	0	

App 19-C

TABLEAU 3:

BASIS	c_j	a_1 $-\delta^H$	a_2^G $-\delta^H\delta^H$	a_B^H $-(1-\delta^H)(1-\nu)\delta^H$	C_B^H $-(1-\delta^H)^2(1-\nu)$	a_B^L $-(1-\delta^H)\nu\delta^L$	S_1	S_2	S_3	S_4	S_5	
a_1	$-\delta^H$	1	$\frac{[(1-\gamma)\delta^H\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{(1-\gamma)(1-\delta^H)(1-\nu)\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	$\frac{(1-\gamma)(1-\delta^H)^2(1-\nu)\beta}{(1-\gamma)\delta^H+\gamma\delta^L}+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)](1-\delta^L)}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$\frac{-1}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	0	$-\frac{(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$\frac{2r}{(1-\gamma)\delta^H+\gamma\delta^L}$
S_2	0	0	$\frac{\delta^L[(1-\mu)\delta^H+\mu\delta^L]-\delta^L[(1-\gamma)\delta^H\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-\frac{\delta^L(1-\gamma)(1-\delta^H)(1-\nu)\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}+\delta^L(1-\delta^L)-\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-\frac{\delta^L(1-\gamma)(1-\delta^H)^2(1-\nu)\beta}{(1-\gamma)\delta^H+\gamma\delta^L}+(1-\delta^L)(1-\delta^L)-\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L(1-\delta^L)}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$\frac{\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	1	0	$-(1-\delta^L)+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$\frac{2r-2\delta^L r}{(1-\gamma)\delta^H+\gamma\delta^L}$
S_3	0	0	0	δ^H	$(1-\delta^H)\beta$	0	0	0	1	0	0	r
a_B^L	$-(1-\delta^H)\nu\delta^L$	0	0	-1	$-(1-\delta^L)/\delta^L$	1	0	0	0	$1/\delta^L$	0	0
S_5	0	0	0	δ^L	$1-\delta^L$	0	0	0	0	-1	1	r
	Z_j											
	C_j-Z_j	0	$-\delta^H\delta^H+\frac{\delta^H[(1-\gamma)\delta^H\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$ ≤ 0	$-(1-\delta^H)(1-\nu)\delta^H+\frac{\delta^H(1-\gamma)(1-\delta^H)(1-\nu)\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}-(1-\delta^H)\nu\delta^L+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	$-(1-\delta^H)^2(1-\nu)+\frac{\delta^H(1-\gamma)(1-\delta^H)^2(1-\nu)\beta}{(1-\gamma)\delta^H+\gamma\delta^L}-(1-\delta^H)\nu(1-\delta^L)+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)](1-\delta^L)\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$\frac{-\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	0	$(1-\delta^H)\nu-\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	

HIP-19 d

TABLEAU 4:

BASIS	C_j	a_1	a_2^G	a_B^H	C_B^H	a_B^L	s_1	s_2	s_3	s_4	s_5	
		$-\delta^H$	$-\delta^M\delta^H$	$-(1-\delta^H)(1-\nu)\delta^H$	$-(1-\delta^H)^2(1-\nu)$	$-(1-\delta^H)\nu\delta^L$						
a_1	$-\delta^H$	1	$\frac{[(1-\gamma)\delta^M\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$[\frac{(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)}{(1-\gamma)\delta^H+\gamma\delta^L}]$ $[(1-\delta^L)-\delta^L(1-\delta^H)B/\delta^H]$	0	$\frac{-1}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$\frac{(1-\gamma)(1-\delta^H)(1-\nu)}{(1-\gamma)\delta^H+\gamma\delta^L}$ $\frac{(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)}{(1-\gamma)\delta^H+\gamma\delta^L}$ δ^L/δ^H	$-\frac{(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$\frac{2r}{(1-\gamma)\delta^H+\gamma\delta^L}-\frac{(1-\gamma)(1-\delta^H)(1-\nu)r}{(1-\gamma)\delta^H+\gamma\delta^L}$ $-\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L r/\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$
s_2	0	0	$\delta^L[(1-\mu)\delta^H+\mu\delta^L]$ $\frac{\delta^L[(1-\gamma)\delta^M\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$(1-\delta^L)^2-\delta^L(1-\delta^L)(1-\delta^H)B/\delta^H$ $-[\frac{(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)}{(1-\gamma)\delta^H+\gamma\delta^L}]$ $[\delta^L(1-\delta^L)-\delta^L\delta^L(1-\delta^H)B/\delta^H]$	0	$\frac{\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	1	$\frac{\delta^L(1-\gamma)(1-\delta^H)(1-\nu)}{(1-\gamma)\delta^H+\gamma\delta^L}$ $-\delta^L(1-\delta^L)/\delta^H+$ $\frac{(1-\gamma)(1-\delta^H)\nu+\gamma(1-\gamma^L)}{(1-\gamma)\delta^H+\gamma\delta^L}$ $\delta^L\delta^L/\delta^H$	$-(1-\delta^L)+$ $\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\gamma^L)]\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$2r-\frac{2\delta^L r}{(1-\gamma)\delta^H+\gamma\delta^L}$ $+\frac{\delta^L(1-\gamma)(1-\delta^H)(1-\nu)r}{(1-\gamma)\delta^H+\gamma\delta^L}-\delta^L(1-\delta^L)r/\delta^H$ $+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L\delta^L/\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$
a_B^H	$(1-\delta^H)(1-\nu)\delta^H$	0	0	1	$(1-\delta^H)B/\delta^H$	0	0	0	$1/\delta^H$	0	0	r/δ^H
a_B^L	$-(1-\delta^H)\nu\delta^L$	0	0	0	$-(1-\delta^L)/\delta^L+(1-\delta^H)B/\delta^H$	1	0	0	$1/\delta^H$	$1/\delta^L$	0	r/δ^H
s_5	0	0	0	0	$(1-\delta^L)-\delta^L(1-\delta^H)B/\delta^H$	0	0	0	$-\delta^L/\delta^H$	-1	1	$r-\delta^L r/\delta^H$
	z_j										0	
	c_j-z_j	0	$-\delta^M\delta^H$ $+\frac{\delta^H[(1-\gamma)\delta^M\delta^H+\gamma\delta^L(1-\mu)\delta^H+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^H+\gamma\delta^L}$ ≤ 0	0	$-(1-\delta^H)^2(1-\nu)(1-B)$ $-(1-\delta^H)\nu\phi$ $+\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\phi\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$ > 0	0	$\frac{-\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$	0	$(1-\delta^H)(1-\nu)$ $-\frac{\delta^H(1-\gamma)(1-\delta^H)(1-\nu)}{(1-\gamma)\delta^H+\gamma\delta^L}$ $+(1-\delta^H)\nu\delta^L/\delta^H$ $-\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^L}{(1-\gamma)\delta^H+\gamma\delta^L}$	$(1-\delta^H)\nu-$ $\frac{[(1-\gamma)(1-\delta^H)\nu+\gamma(1-\delta^L)]\delta^H}{(1-\gamma)\delta^H+\gamma\delta^L}$		

Notice the C_B^H column. The factor $(1-\gamma^L)-\delta^L(1-\delta^H)B/\delta^H=\phi$

APP-19e

TABLEAU 5:

BASIS	c_j	a_1	a_2^C	a_B^H	c_B^H	a_B^L	s_1	s_2	s_3	s_4	s_5	
		$-\delta^M$	$-\delta^M\delta^M$	$-(1-\delta^M)(1-\nu)\delta^M$	$-(1-\delta^M)^2(1-\nu)$	$-(1-\delta^M)\nu\delta^L$	0	0	0	0	0	
a_1	$-\delta^M$	1	$\frac{[(1-\gamma)\delta^M\delta^M+\gamma\delta^L(1-\mu)\delta^M+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^M+\gamma\delta^L}$	0	0	0	$\frac{1}{(1-\gamma)\delta^M+\gamma\delta^L}$	0	$\frac{(1-\gamma)(1-\delta^M)(1-\nu)}{(1-\gamma)\delta^M+\gamma\delta^L}$	0	$-\frac{(1-\gamma)(1-\delta^M)\nu+\gamma(1-\delta^L)}{(1-\gamma)\delta^M+\gamma\delta^L}$	$-\frac{[(1-\gamma)(1-\delta^M)+\gamma(1-\delta^L)]r}{(1-\gamma)\delta^M+\gamma\delta^L}+\frac{2r}{(1-\gamma)\delta^M+\gamma\delta^L}$
s_2	0	0	$\delta^L[(1-\mu)\delta^M+\mu\delta^L]-\frac{\delta^L[(1-\gamma)\delta^M\delta^M+\gamma\delta^L(1-\mu)\delta^M+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^M+\gamma\delta^L}$	0	0	0	$\frac{\delta^L}{(1-\gamma)\delta^M+\gamma\delta^L}$	1	$\frac{\delta^L(1-\gamma)(1-\delta^M)(1-\nu)}{(1-\gamma)\delta^M+\gamma\delta^L}$	0	$-(1-\delta^L)+\frac{[(1-\gamma)(1-\delta^M)\nu+\gamma(1-\delta^L)]\delta^L}{(1-\gamma)\delta^M+\gamma\delta^L}$	$2r-\frac{2\delta^L r}{(1-\gamma)\delta^M+\gamma\delta^L}-(1-\delta^L)r+\frac{\delta^L[(1-\gamma)(1-\delta^M)+\gamma(1-\delta^L)]r}{(1-\gamma)\delta^M+\gamma\delta^L}$
a_B^H	$-(1-\delta^M)(1-\nu)\delta^M$	0	0	1	0	0	0	0	$\frac{1-\delta^L}{\delta^M\phi}$	$\frac{(1-\delta^M)\theta}{\phi\delta^M}$	$\frac{-(1-\delta^M)\theta}{\phi\delta^M}$	$r/\delta^M-\frac{(1-\delta^M)\theta}{\phi\delta^M}[r-\delta^L r/\delta^M]$
a_B^L	$-(1-\delta^M)\nu\delta^L$	0	0	0	0	1	0	0	0	0	$1/\delta^L$	r/δ^L
c_B^H	$-(1-\delta^M)^2(1-\nu)$	0	0	0	1	0	0	0	$\frac{-\delta^L}{\phi\delta^M}$	$-1/\phi$	$1/\phi$	$\frac{(\delta^M-\delta^L)r}{\phi\delta^M}$
	z_j											
	c_j-z_j	0	$-\delta^M\delta^M+\frac{\delta^M[(1-\gamma)\delta^M\delta^M+\gamma\delta^L(1-\mu)\delta^M+\gamma\delta^L\mu\delta^L]}{(1-\gamma)\delta^M+\gamma\delta^L}$ ≤ 0	0	0	0	$\frac{\delta^M}{(1-\gamma)\delta^M+\gamma\delta^L}$	0	$(1-\delta^M)(1-\nu)-\frac{\delta^M(1-\gamma)(1-\delta^M)(1-\nu)}{(1-\gamma)\delta^M+\gamma\delta^L}-\frac{(1-\delta^M)^2(1-\nu)(1-\theta)\delta^L}{\phi\delta^M}$		$-\frac{(1-\delta^M)^2(1-\nu)(1-\theta)}{\phi}$	$\frac{(1-\delta^M)^2(1-\nu)(1-\theta)}{\phi}+(1-\delta^M)\nu-\frac{\delta^M[(1-\gamma)(1-\delta^M)\nu+\gamma(1-\delta^L)]}{(1-\gamma)\delta^M+\gamma\delta^L}$

$G_{15} \equiv [1-\gamma]\bar{\delta}^H \nu + \gamma \bar{\delta}^L$.

We now present the successive Simplex tableaus.

[INSERT TABLEAUS ABOUT HERE]

Tableau 5 is the end-tableau if all values in the last column are non-negative and if all $C_j - Z_j$ values are non-negative. Except for the $C_j - Z_j$ value in the S_5 column, these are easy to verify. From Tableau 5, we get

$$[C_j - Z_j]\Big|_{j = S_5} = (\delta^L)^{-1}\{-G_{15}\delta^L\delta^H[(1-\gamma)\delta^H + \gamma\delta^L]^{-1} + G_{16} + \bar{\delta}^H \nu \delta^L\} \qquad (A-45).$$

where $G_{16} \equiv [\bar{\delta}^H]^2[1-\nu][1-\beta]\delta^L \phi^{-1}$.

The expression in (A-45) can be positive or negative. Before writing down a parametric restriction that fixes the sign of (A-45), we will give a derivation which will result in (A-45) and has a significant economic meaning. First, note that the solution in Tableau 5 indicates that bad borrowers at t=0 get a better than first best contract over their credit horizon (the expression for S_2 in Tableau 5 is non-negative: see also (30) and (30)'). This better-than-first-best contract is at the expense of good borrowers at t=0. Thus, we can interpret the maximization procedure stated above as searching for the solution that minimizes the premium bad borrowers at t=0 get over their first best contract. Therefore, in an optimal solution one tries to give the maximum feasible reward to borrowers with good first period realizations (see the result $\alpha_2^G = 0$), and impose the maximum feasible penalty on borrowers with bad first period realizations (see the result that they get spot market separating contracts). This is the solution characterized in Tableau 5. However, one other solution exists. That is because the contracts offered to unsuccessful first period types are separating (see Figure 2), which implies that some deadweight costs, related to collateral, are associated with those contracts.

Look at (35) to see that reducing α_B^L enables the banks to decrease C_B^H. The following diagram of perturbations is feasible.

$$\Delta\alpha_B^L\downarrow \longrightarrow \begin{cases} \Delta C_B^H\downarrow \\ \Delta\alpha_B^H\uparrow \end{cases} \text{follows from the binding constraints (32) and (35)}$$

$$\Delta\alpha_B^L\downarrow \searrow \Delta\alpha_I\uparrow \text{ follows from the binding intertemporal profit condition (29).}$$

It follows from the binding constraints (29), (32), (33), and (35) that for all ϵ sufficiently small, the following set of perturbations hold

$$\begin{aligned} &\Delta\alpha_B^L = -\epsilon,\ \Delta\alpha_1 = G_{15}\delta^L\epsilon[\{1-\gamma\}\delta^H + \gamma\delta^L]^{-1}, \\ &\Delta C_B^H = -\ \delta^L\epsilon\phi^{-1},\ \Delta\alpha_B^H = \bar{\delta}^H\delta^L\beta\epsilon[\phi\delta^H]^{-1}. \end{aligned} \tag{A-46}$$

The effect of this set of perturbations on the objective function (28) is

$$\Delta\mathcal{F} = \epsilon\{-G_{15}\delta^L\delta^H\{[1-\gamma]\delta^H + \gamma\delta^L\}^{-1} + G_{16} - \delta^H\nu\delta^L\}. \tag{A-47}$$

Compare (A-47) with (A-45) to see that our assumption $C_j - Z_j \Big|_{j = S_5} \leq 0$ in Tableaus 1 through 5 is identical to assuming that it is not optimal to give unsecured loans in the second period to borrowers with bad first period realizations. We can now distinguish two solutions. The first solution holds when we assume that the expression for $C_j - Z_j \Big|_{j = S_5}$ given in (A-45) is non-positive. This solution can be found in Tableau 5. The alternative solution is based on the assumption that $C_j - Z_j \Big|_{j = S_5}$ is positive. Note that, since the model is linear, ϵ will be chosen such that the perturbations in (A-46) completely eliminate collateral. This implies

$$\bar{\bar{C}}_B^H = \bar{C}_B^H + \Delta C_B^{H*} = 0,$$

or equivalently,

$$\delta r[\phi\delta^H]^{-1} - \delta^L\epsilon^*\phi^{-1} = 0,$$

which means

$$\varepsilon^* = \delta r[\delta^H \delta^L]^{-1}.$$

This gives us

$$\Delta\alpha_1^* = G_{15}\delta r\{\delta^H[(1-\gamma)\delta^H + \gamma\delta^L]\}^{-1},$$
$$\Delta\alpha_B^{L*} = \delta r[\delta^H \delta^L]^{-1}$$
$$\Delta\alpha_B^{H*} = \bar{\delta}^H \delta\beta r[\phi(\delta^H)^2]^{-1},$$
$$\Delta C_B^{H*} = 0.$$

Subtract the perturbations listed above from the single bar, "$-$", solution in Tableau 5 (or in the theorem), to get the double bar, "$=$", solution also given in the theorem. Q.E.D.

<u>PROOF OF THEOREM 6</u>: <u>-cf(A)</u>. The (separating) spot market equilibrium exists if it can not be broken by a pooling alternative. The pooling alternative consists also of single period contracts. As in the intertemporal market equilibrium case (see Figure 2) it is also partly separating. As a matter of fact, the pool of successful borrowers as well as the pool of unsuccessful borrowers can be offered separating contracts. In that case these contracts are optimally collateralized contracts. Schematically we get,

<u>t=0</u> <u>t=1</u>

Pooling contract $\alpha_1 = r/\bar{\delta}$
where, $\bar{\delta} \equiv [1-\gamma]\delta^H + \gamma\delta^L$

$\xrightarrow{\bar{\delta}}$ standard optimally collateralized separating contract
$\alpha_G^H = r[\delta^H]^{-1} - \bar{\delta}^H \beta C_G^H [\delta^H]^{-1}$
$C_G^H = \delta r[\phi\delta^H]^{-1}$, $\alpha_G^L = r[\delta^L]^{-1}$.

$\xrightarrow{1-\bar{\delta}}$ idem:
$\alpha_B^H = r[\delta^H]^{-1} - \bar{\delta}^H \beta C_B^H [\delta^H]^{-1}$
$C_B^H = \delta r[\phi\delta^H]^{-1}$, $\alpha_B^L = r[\delta^L]^{-1}$.

Note that $\{\alpha_G^H, C_G^H, \alpha_G^L\}$ is separating if

$$\overline{\delta}^H[1-\beta][\phi\delta^H]^{-1} \leq \tau\{[1-\tau]\delta^H + \tau\delta^L\}^{-1}, \tag{A-48}$$

where $\tau \equiv \gamma\delta^L\mu\{\gamma\delta^L + [1-\gamma]\delta^H\}^{-1}$.

The expression in (A-48) is a standard single period existence restriction. Proofs can be found in Lemma 5. The derivation for τ follows from Figure 2. The existence restriction for the contract $\{\alpha_B^H, C_B^H, \alpha_B^L\}$ is derived in Lemma 5. However, the proof in Lemma 5 indicates that this restriction will not be binding if the spot market Nash equilibrium exists. Proving existence requires showing that the utility of a good borrower under the single period contract solution in Theorem 2 exceeds its utility under the pooling contract solution above. Substitute the Theorem 2 solution in the definition of $U_I(\delta^H|\delta^H)$ and use (3) to get

$$\begin{aligned} U_I(\delta^H|\delta^H) = {} & \delta^H R_N^H - [1-\beta]\overline{\delta}^H\hat{C}_I^H + [\delta^H]^2 R_N^H \\ & - \overline{\delta}^H[1-\nu]\hat{\pi}_{III}^H[\delta^H R_N^H - \overline{\delta}^H(1-\beta)W] + \overline{\delta}^H\nu\delta^L R_N^L \end{aligned} \tag{A-49}$$

The pooling alternative in the diagram above leads to.

$$\begin{aligned} \overline{U}_I(\delta^H) = {} & \delta^H[R-r(\overline{\delta})^{-1}] + \{\delta^H + \overline{\delta}^H[1-\nu]\}\{\delta^H R_N^H - [1-\beta]\overline{\delta}^H\delta r[\phi\delta^H]^{-1}\} \\ & - \overline{\delta}^H\nu\delta^L R_N^L \end{aligned} \tag{A-50}$$

Existence is guaranteed if $U_I(\delta^H|\delta^H) \geq \overline{U}_I(\delta^H)$. This results directly in condition (38). Condition (39) is straightforward. This condition guarantees the existence of the separating contracts in the contract III nodes. The LHS is the borrower's utility under the pooling contract. The RHS gives the borrower's utility under the separating contract. Condition (40) is a similar condition for the contract IV nodes. For its derivation, see Lemma 5.

-cf(B). The good borrower's utility under the intertemporal market equilibrium in Theorem 4 should exceed its utility under the pooling contract equilibrium in Theorem 5. Given risk neutrality and the absence of rationing, this is the same as requiring the borrowing cost for a good borrower to be lower in the intertemporal contract solution in Theorem 4 than in the pooling solution in Theorem 5. The borrowing cost in Theorem 4 is

$$\delta^H \tilde{\alpha}^H_I + \bar{\delta}^H \tilde{C}^H_I + \bar{\delta}^H [1-\nu][\delta^H \tilde{\alpha}^H_{III} + \bar{\delta}^H \tilde{C}^H_{III}] + \bar{\delta}^H \nu \delta^L \tilde{\alpha}^L_{III} \tag{A-51}$$

The borrowing cost in Theorem 5 is

$$\delta^H \bar{\alpha}_1 + \bar{\delta}^H [1-\nu][\delta^H \bar{\alpha}^H_B + \bar{\delta}^H \bar{C}^H_B] + \bar{\delta}^H \nu \delta^L \bar{\alpha}^H_B. \tag{A-52}$$

Existence is guaranteed if (A-51) $\leq$ (A-52). This results directly in condition (41). Condition (42) is derived in Theorem 3. and explicitly assumed in Theorem 4. The assumption (43) is of no special value. It is just to indicate that we focus on the pooling contract solution in (36).

All that remains is to show that the set of parameter values for which (38) - (43) and (A-48) are satisfied is non-empty. It can be easily verified that the following parameter values achieve this.

$\delta^H = .75$	implying	$\phi = .35$
$\delta^L = .5$		$\xi = .416$
$\gamma = \nu = \mu = .5$		$\tau = .2$ (see Figure 2)
$\beta = .9$		$\hat{C}^H_I = 1.3282313$
$r = 1.1$		$\hat{\pi}^H_{III} = .8968184$
$R = 4$		$\theta = .9750652$ (follows from $\tilde{C}^H_I + \tilde{C}^H_{III} \equiv W_0$)
$W_0 = 2.08$		

Q.E.D.

LIST OF KEY SYMBOLS

δ^L = the success probability for a bad borrower;

δ^H = the success probability for a good borrower;

π_j^i = the credit granting probability in the set of contracts j for borrower type i ($i \in \{L, H\}$, $j \in \{I, II, III, IV, V\}$);

α_j^i = the interest factor (= one plus the interest rate) in the set of contracts j for borrower type i($i \in \{L, H\}$, $j \in \{I, II, III, IV, V\}$);

β = measure of the bank's evaluation of a borrower's collateral. That is, \$1 collateral has a value of β to the bank;

γ $(1-\gamma)$ = the proportion of bad (good) borrowers at t=0;

η $(1-\eta)$ = " " " in the set of nodes II;

ν $(1-\nu)$ = " " " " " III;

μ $(1-\mu)$ = " " " " " IV;

σ $(1-\sigma)$ = " " " " " V;

Ψ $(1-\Psi)$ = the proportion of bad (good) borrowers at t=1 within the pool of borrowers rationed at t=0;

Ω $(1-\Omega)$ = the proportion of bad (good) borrowers within the pool of borrowers with bad first period realizations;

τ $(1-\tau)$ = the proportion of bad (good) borrowers within the pool of borrowers with good first period realizations;

λ = Lagrange multiplier;

ρ = decay parameter for delayed investment projects;

Θ = the shortage of collateral parameter, $\Theta \in [0, 1)$; if $\Theta = 1$, no shortage applies;

$\delta^G \equiv [1-\tau]\delta^H + \delta^L$

$\bar{\delta} \equiv [1-\gamma]\delta^H + \gamma\delta^L$

$\phi \equiv (\delta^H[1-\delta^L] - \beta\delta^L[1-\delta^H])(\delta^H)^{-1}$

$\xi \equiv (\delta^H[1-\delta^L] - \nu\delta^L[1-\delta^H])(\delta^H)^{-1}$

$C_{III}^H(\text{opt}) \equiv [\delta^H - \delta^L]r[\phi\delta^H]^{-1}$ = the level of collateral in an optimally collateralized single period contract;

R = the return on the investment project if successful;

r = the risk free interest factor (= one plus the risk free interest rate);

W_0 = the initial (t=0) level of available collateral for each individual borrower;

$W \equiv W_0 - C_I$ = the available collateral in the second period if C_I has been lost in the first period;

C_j^i = the collateral asked in the set of contracts j from a type i borrower ($i \in \{L, H\}$; $\gamma \in \{I, II, III, IV, V\}$);

$U_j(\delta^k | \delta^i)$ = the expected ability for a type i borrower who chooses the type k contracts, starting from the set of nodes j;

α_1 = the first period pooling interest rate;

α_2^G = the second period pooling interest rate conditioned on a good first period realization;

$\delta \equiv \delta^H - \delta^L$

$\bar{\delta}^i \equiv 1 - \delta^i$, $i \in \{H, L\}$

$R_N^j \equiv R - r[\delta^j]^{-1}$, $j \in \{H, L\}$

$\mathfrak{B}^i$ ≡ dynamic strategic credit policy of bank i

$\mathfrak{B}_1$ ≡ first period credit policy

$\mathfrak{B}_2(y_1, \chi_1)$ ≡ second period credit policy applicable to borrower with first period contract choice y_1 and first period realization χ_1.

N ≡ set of all possible competing banks (there are n banks)

ζ_i ≡ net expected profit of bank i

κ_1 ≡ borrower's first period type

κ_2 ≡ borrower's second period type

$z \equiv (\kappa_1, \chi_1, \kappa_2)$ is borrower's composite type

y_2 ≡ borrower's second period contract choice

$\mathcal{F}$ ≡ bank's objective function

$\mathcal{P}$ ≡ Lagrangian

α_2^B = the second period pooling interest rate conditioned on a bad first period realization:

In Lemma 5 it is established that the bad return pool within an intertemporal pooling contract gets a separating contact. The following variables are defined for that case.

α_B^L = second period interest factor for bad types in the pool of unsuccessful first period borrowers;

α_B^H = second period interest factor for good types in the pool of unsuccessful first period borrowers;

C_B = second period collateral asked from a good types in the pool of unsuccessful first period borrowers;

Some additional symbols:

"^" on top of variables indicates the (separating) single-period-contract solution:

"~" on top of variables indicates the (separating) intertemporal contract solution:

"—" or "=" on top of variables indicates the (intertemporal) pooling contract solution.

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